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(Residential Autonomous College under University of Calcutta)

B.A./B.SC. THIRD SEMESTER EXAMINATION, DECEMBER 2011

SECOND YEAR

Date : 22/12/2011 **MATHEMATICS (General)**

Time : 11 am – 2 pm Paper : III Full Marks : 75

[Use separate Answer Books for each group]

Group – A

1. Answer **any four** questions : [5×4 = 20]

   a) Prove that the equation of the plane which passes through (1, –2, 4) and (3, –4, 5) and parallel to the x-axis is y + 2z = 6 [5]

   b) A variable plane is always at a constant distance p from the origin and meets the co-ordinate axes at A, B, C. Prove that the locus of the point of intersection of the three planes through A, B, C parallel to the co-ordinate planes is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$. [5]

   c) Find the shortest distance between the straight lines $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ and $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$. Find also the equations of the line of shortest distance. [5]

   d) Find the equation of the sphere having its centre on the line 5y + 2z = 0 = 2x – 3y and passing through the two points (0, –2, –4) and (2, –1, –1). [5]

   e) Show that the plane 2x + y –z = 12 touches the sphere $x^2 + y^2 + z^2 = 24$ and find the point of contact. [2+3]

   f) Find the equation of the right circular cone whose vertex is the origin, axis is the y-axis and semi-vertical angle is 60°. [5]

Group – B

2. Answer **any four** questions : [2×4 = 8]

   a) Define extreme point of a convex set. [2]

   b) Introduce slack and surplus variables to convert the system into standard form :

   Maximize $z = 2x_1 + x_2$

   subject to $x_1 \le 4$

   $2x_1 + x_2 \ge 1$

   and, $x_1, x_2 \ge 0$ [2]

   c) Express the vector (1,1,1) as a linear combination of the vectors (1,2,3), (4,2,1) and (2, 4, 2) in E^3. [2]

   d) Find graphically the feasible space for the L.P.P :

   Maximize $z = -2x_1 + 5x_2$

   Subject to $5x_1 + 2x_2 \le 20$

   $4x_1 + 5x_2 \le 10$

   $x_1 \ge 2,\ x_2 \ge 0$ [2]

   e) What is the difference between a basic solution and a degenerate basic solution? [2]

f) Find the initial basic feasible solution of the following transportation problem by North-West corner rule : [2]

	D_1	D_2	D_3	A_i
O_1	4	6	9	10
O_2	4	2	7	15
B_j	8	7	10	

g) Can assignment problem be treated as a linear programming problem? Justify. [2]

Answer **any two** questions from Q. No. 3 – 6 : [16×2]

3. a) A farmer buys sheep and goats at Rs. 80 per sheep and at Rs. 100 per goat, and sells them at a profit of Rs. 10 per sheep and Rs. 15 per goat. The farmer has an accommodation for not more than 50 animals and cannot afford to pay more than Rs. 4400. He wishes to buy these two kinds of animals in order to have the maximum profit. Pose an LPP for this problem. [8]

b) Solve graphically the following LPP :

$$\text{Minimize} \quad z = -x_1 + 2x_2$$
$$\text{subject to} \quad -x_1 + 3x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$\text{and} \quad x_1, x_2 \ge 0$$

[8]

4. a) Reduce the feasible solution (1, 1, 1) of the set of equations $x_1 + x_2 + 2x_3 = 4$, $2x_1 - x_2 + x_3 = 2$ to a basic feasible solution. [8]

b) Write down the dual of the following primal problem :

$$\text{Minimize} \quad z = x_1 + 3x_2$$
$$\text{subject to} \quad x_1 + x_2 \le 3$$
$$2x_1 - x_2 \ge -1$$
$$x_1 + 2x_2 = 5$$

Where $x_1 \ge 0$, x_2 is unrestricted in sign. [8]

5. a) Use Charne's Big-M method to solve the following L.P.P : [8]

$$\text{Maximize} \quad z = 3x_1 - x_2$$
$$\text{subject to} \quad -x_1 + x_2 \ge 2$$
$$5x_1 - 2x_2 \ge 2$$
$$x_1, x_2 \ge 0$$

b) Solve the assignment problem given by the following cost-matrix : [8]

	a	b	c	d
1	18	17	12	11
2	19	15	11	16
3	25	21	17	11
4	16	14	11	11

6. a) Use simplex method to solve the L.P.P [6]

Maximize $z = 4x_1 + 10x_2$

subject to $2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$

and $x_1, x_2 \ge 0$

b) Obtain an initial B.F.S to the following transportation problem Using N-W corner method : [2]

	D_1	D_2	D_3	
O_1	3	2	5	6
O_2	9	1	2	10
O_3	4	3	1	12
	9	16	3	

c) Obtain an optimal basic feasible solution to the following transportation problem : [8]

	D_1	D_2	D_3	D_4	availability
O_1	12	10	13	11	28
O_2	9	11	14	15	22
O_3	16	13	12	15	10
Requirements :	10	14	19	17	

Group – C

7. Answer **any three** questions : [5×3 = 15]

a) i) Round-off 0·0056812 and 40·3685 upto four significant digits. [1]

ii) Write down the approximate representation of $\frac{2}{3}$ correct upto four decimal place and then find absolute, relative and percentage errors. [4]

b) i) If $\pi = 3{\cdot}14$ is used in place of 3·14156, find the absolute and relative errors. [1]

ii) Given :

x	1·00	1·10	1·20	1·30
f(x)	0·8415	0·8912	0·9320	0·9626

compute f(1·02) [4]

c) Use the method of bisection to find the root of the equation $x^4 + 2x^3 - x - 1 = 0$ lying in the interval (0,1) correct upto 3 decimal places. [5]

d) Find the approximate value of $\int_0^1 \frac{x\,dx}{1+x^2}$ upto four places of decimal by Simpson's $\frac{1}{3}$ rule taking 6 equal sub-intervals of [0,1] and hence find the approximate value of log 2 correct to four places of decimal. [4+1]

e) Using Newton-Raphson method, find a real root of the equation $x^2 - 5x + 2 = 0$ correct to three places of decimals. [5]